

MEASUREMENT OF VIBRATION TRANSMISSIBILITY FOR CONTINUOUS SYSTEMS

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ABSTRACT

Vibration transmissibility express the output to input vibration relationship of a vibrating system. It is normally used in vibration isolation studies where the formulation for an one degree-of-freedom system (1-DOF) is generally employed. Nevertheless, transmissibility should not be limited to isolator studies and it may be necessary to quantify transmissibility between two specific points of interest when there is a continuous system between them. There is few or almost none literature explaining how to measure transmissibility. Frequency Response Function between the output and input of the system is normally used. However, no mention is made to what type of signal processing is performed to obtain such a function. Depending on the signal processing used, completely different results are obtained. To show that is exactly the main objective of this paper. Several types of functions were tested and the results are shown with the respective comments and considerations.

NOMENCLATURE

$\bar{}$	= complex conjugate of a function
BW	= window bandwidth
crt_n	= correction factor
$C_{spec_{x,y}}$	= cross spectrum between points x and y
FFT	= Fast Fourier Transform
FRF	= Frequency Response Function
L_{spec_y}	= linear spectrum of point y
N	= number of averages used
P_{spec_y}	= power spectrum of point y
r	= frequency ratio (ω/ω_n)
sum	= sum of values
$Tr_{x,y}$	= Transmissibility Function between points x and y
$Wtime$	= widowed time
x	= measurement point x

$*$	= multiplication
X	= Response Function at point x (freq.)
X_{avg}	= exponential averaged value
y	= measurement point y
Y	= Response Function at point y (freq.)
n	= number of the measurement
ω	= Forcing (or excitation) frequency
ω_n	= natural frequency of the isolator
ζ	= viscous damping ratio

1. INTRODUCTION

Vibration isolation is the amount of vibration that is not transmitted to other components. Therefore, isolation is directly related to transmissibility studies. Although there is a lot of literature related to its formulation, there is few related to its measurement.

According to reference [10], isolation systems are comprised by three subsystems: the object to be isolated, the supporting structure (floor, foundation), and the vibration isolators (mounts) placed between them. Depending on the situation, it may be necessary to isolate the object from the supporting structure or the supporting structure from the object. The formulation used for both situations is the same. Although each of the subsystems mentioned represents a multi-degree-of-freedom, in fact, only the lowest structural modes of the object and the supporting structure are critical for the effectiveness of the isolator and so, they can be considered to be ideal rigid bodies. Moreover, usually the mounts' mass are small compared to the mass of the object and so, the isolator can be considered a massless spring [10].

The transmissibility formulation used in isolation studies is normally the one developed for a 1-DOF system. However, it may be necessary to obtain transmissibility when there is more than one degree-of-freedom involved. The literature

found for this case is mainly that related to ODS (Operational Deflection Shape) studies, which needs the measurement of the operating acceleration at a series of points relative to the acceleration at a reference point [4, 6]. That can be considered also to be a transmissibility. The point here is that, although some of these studies mention some signal processing aspects involved, they do not point out what will be the consequences of an improper choice of it.

2. TRANSMISSIBILITY

2.1 Transmissibility Formulation

As mentioned, transmissibility is the relationship between the output to input responses of a vibration system, as follows:

$$Tr_{x,y} = \frac{\text{output}}{\text{input}} = \frac{X}{Y} = \frac{F_T}{F_0} \quad (1)$$

In equation (1), relationship X/Y is normally used when the source of vibration is the foundation and relationship F_T/F_0 is used when the source of vibration is the machine (or object).

The 1-DOF theory assumes that the only natural frequency present is the one related to the isolator, since the machine and the foundation, as mentioned previously, are considered as rigid bodies and the isolator is assumed massless [1].

The absolute transmissibility for a 1-DOF system with viscous damping can be calculated as [1, 9]:

$$Tr_{x,y} = \frac{X}{Y} = \frac{F_T}{F_0} = \frac{1 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}} \quad (2)$$

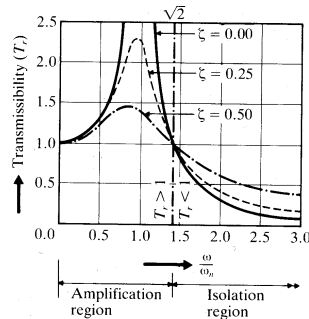


Figure 1 – Transmissibility for a 1-DOF system with viscous damper [9]

Figure 1 shows a graphical representation of equation (2). By calling the relationship $\omega/\omega_n = r$, it is possible to observe that transmissibility reaches its maximum value at $r = 1$. In the region where $0 < \omega < \sqrt{2}\omega_n$ vibration is amplified, whereas in the region $\omega > \sqrt{2}\omega_n$, vibration is attenuated (or isolated). It can be noticed by analysing equation (2) and Figure 1 that the transmissibility value is very much influenced by the amount of viscous damping present in the system. The increase of damping have different effects according to the region under consideration.

When one is designing an isolator, ω needs to be in the isolation region and so, by increasing the damping ratio (ζ), more vibration is transmitted. So, for practical purposes, the isolators are normally calculated without damping, using the following formula and some provision to damping is made afterwards:

$$Tr_{x,y} = \frac{X}{Y} = \frac{F_T}{F_0} = \frac{1}{(r^2 - 1)} \quad (3)$$

In equations (2) or (3), if there is more than one forcing frequency, the one to be considered for the isolator is the smallest value present in the isolation region, since by increasing its value, the isolation increases (see Figure 1). The natural frequency in this case is the isolator natural frequency, where the stiffness is that of the isolator and the mass is that of the object to be isolated.

Considering now the case when there is a continuous system, instead of this 1-DOF theory, there will not be only one, but infinite natural frequencies for the system. However, by analysing equations (2) or (3) and Figure 1, one can say that only if the forcing frequency coincides with one of the natural frequencies of the continuous system there will be a problem with resonance, therefore, high transmissibility values. So, one important point to make here is that, for a continuous system, transmissibility needs to be considered only at the forcing frequencies. These are normally, the harmonics of the turning speed of the equipment.

2.2 Transfer Function Measurement

The point of this paper is: How to measure transmissibility? Following equation (1), it can be concluded that transmissibility can be measured in the same way as Frequency Response Functions (FRFs) are calculated in Modal Analysis. However, now, instead of using a force function as input, both input and output will be response

functions (due to the forces applied), measured at different points of the structure. The forcing frequencies to be considered in equation (1) can be obtained by measuring the spectra values of the input in any of the available formats as it will be presented below (i.e., cross or auto spectra).

FRFs are known sometimes as transfer functions [2], in the same way as Transmissibility is. Transfer functions $H(f)$ describes both the magnitude and phase of the response input as a function of the input frequency. Therefore, it is a complex quantity. There are several relationships governing the process, i.e. [2]:

2.2.1 Fourier transforms:

$$S_x(f) = S_y(f)H(f) \quad \text{or} \quad H(f) = \frac{S_x(f)}{S_y(f)} \quad (4)$$

2.2.2 Auto Spectral Densities:

$$G_{xx}(f) = G_{yy}(f)|H(f)|^2 \quad \text{or} \quad |H(f)|^2 = \frac{G_{xx}(f)}{G_{yy}(f)} \quad (5)$$

where:

$$G_{aa}(f) = S_a(f)\overline{S_a(f)} = |S_a(f)|^2 \quad (6)$$

with a used to represent each channel (i.e., either x or y) above.

2.2.3 Cross Spectral Densities:

$$G_{xy}(f) = G_{yy}(f)H_1(f) \quad \text{or} \quad H_1(f) = \frac{G_{xy}(f)}{G_{yy}(f)} \quad (7)$$

$$G_{xx}(f) = G_{yx}(f)H_2(f) \quad \text{or} \quad H_2(f) = \frac{G_{xx}(f)}{G_{yx}(f)} \quad (8)$$

where:

$$G_{xy}(f) = S_x(f)\overline{S_y(f)} \quad \text{and} \quad G_{yx}(f) = S_y(f)\overline{S_x(f)} \quad (9)$$

a and b represents channels x and y or vice-versa in equations (7) and (8), respectively.

The relationships (7) and (8) assumes that there is no noise present in either $x(t)$ or $y(t)$ so that H_1 is equal H_2 . The transfer function obtained by equation (5) can only provide the magnitude information because of the square.

Coherence function should also be computed as it is a measure of the quality of the input response and cross spectral densities, and the causality of input to response [2], as follows:

$$\gamma_{xy}^2 = \frac{|G_{yx}(f)|^2}{G_{yy}(f)G_{xx}(f)} \quad (10)$$

It can be seen that if the signal is noise free, equation (10) should be 1. If coherence value is less than one, indicates that the response is not attributable to the input, due to probably noise or nonlinearity of the system or unexpected input signals [2]. Coherence values range from 0 to 1.

Normally, commercial available analysers compute the H_1 estimator, together with γ_{xy}^2 , equations (7) and (10), respectively [5].

It can be seen by the above equations that transmissibility should then be calculated in the same way as expressed by equation (7):

$$T_{x,y}(f) = \frac{G_{xy}(f)}{G_{yy}(f)} \quad (11)$$

In the case of modal analysis, the resonance frequencies are found by peaks in, at least, some of the FRFs. With transmissibility, this is not the case. Frequencies at which the vibration energy is concentrated are indicated by the cross and auto spectra used in equation (11). However, when they are divided by the other, the peaks cancel out and become flat [6]. So, the forcing frequencies of interest to be considered in equations (2) or (3) will be in fact, the frequencies where the transmissibility is flat (i.e., the frequencies where coherence is maximum), as shown in item 5.

The quantities expressed from equation (4) to (8) are calculated in the HP35670A analyser available in the GRAVI (Group of Acoustics and Vibration) Laboratory according to the type of average used. The main reason for the results presented here is linked with that. Depending on the type of average available, the results are different. Moreover, can one say that (by analysing equation (1)) transmissibility is just the ratio of two measured responses (output to input)?

3. TYPES OF AVERAGES (HP35670A)

In order to evaluate transmissibility, several functions available in the HP35670A analyser were tested in order to compare the calculations performed and to decide with signal processing function should be used for this case.

Before going to the signal processing calculations itself, it is necessary to show the types of averages available. Depending on the type of average used, the calculations performed to convert the measured signal from the time to the frequency domain will be completely different. Next, the average types will be explained.

Averaging and windowing are techniques used to improve the accuracy of the measurement. Av-

eraging a measurement reduces the statistical variance of a measurement with a random excitation function [7].

3.1 Time averaging

According to reference [8], time averaging is normally used during mechanical applications measurements to resolve low-level frequency components from background noise. Although this time of averaging has a better signal-to-noise ratio than RMS averaging, there are some restrictions: 1) the input signal must be periodic; 2) one needs to provide a trigger signal. If the trigger is not provided, the analyser will still makes the measurement but the amplitude of periodic signals will diminish with each successive average.

With time averaging, the analyser averages complex values point by point in the frequency domain. This process lowers noise because the real and imaginary components of the random signals are not in phase and cancel each other. Frequency components that are not periodic do not cancel and therefore do not diminish with successive averages.

This type of averaging was used in this paper since the transfer function calculated using it is just the ratio of the response measured by each channel (see 4.5.2).

3.2 Time exponential averaging

Unlike the linear averaging explained above, exponential averaging weights new data more than old data. This is useful for tracking data that changes over time [8]. For exponential averaging, the number of averages one selects determines the weighting of old versus new values (not the total number of averages one calculates). As one increases the number of averages, new data weighted less. The formula used in this case is presented below, with N = weighting factor:

$$X_{avg} = Y_{n+1} = \frac{1}{N} Y_n + \left(\frac{N-1}{N} \right) Y_{n-1} \quad (12)$$

With exponential averaging, it is important to set correctly the number of averages. If there are too few averages, the averaging will not smooth out variances. However, if there are too many averages, the analyser may not track subtle changes occurring with the data [8].

3.3 RMS (Root-mean-square) averaging

RMS averaging is sometimes called “power” averaging. It is calculated by the square root of

the sum of all values squared, divided by the number of the measurements (mean). It is a good technique for determining the average power level. Because of that, it is the type of default average normally performed by most vibration analysers. It averages N time records as follows:

$$Y_{n+1} = \frac{Y_n + Y_{n-1}}{2} \quad (13)$$

RMS averaging does not eliminate noise present in the signal. In fact, it simply approximates the actual noise level. By increasing its number, a better statistical approximation of noise is obtained, although the noise is not actually reduced.

It is available only for power spectrum, cross spectrum, FRF and coherence calculations. Linear spectrum and time signals only shows the last processed time record.

3.4 RMS exponential

This type of average works in the same way as time exponential averaging, i.e., the new data is weighted more than the old one. It is useful for tracking data that changes over time. Until N averages are reached, there is no difference between RMS exponential and RMS. The number of averages (N) in this case do not indicate the number of averages calculated, yet, determines the weighting of old versus new data. Therefore, by increasing N , new data is weighted less. The formulation used in this case is the same as equation (12).

3.5 Peak hold averaging

The peak hold averaging is not strictly an averaging process. It simply gets the maximum peak at each frequency during each measurement and assumes that value to be the averaged value.

4. SIGNAL PROCESSING FROM TIME TO FREQUENCY DOMAIN (HP35670A)

The functions shown in section 2.2 [2, 3] are calculated by the HP35670A analyser differently according to the type of average used and shown in section 3. The main difference between the formulas presented here and in section 2.2 is the correction factor for each channel, without each the analyser does not guarantee the manufacture values [8]. The available signal processing are:

4.1 Power Spectrum (or Auto Spectra)

This function is sometimes called “Auto Spectra”. It is used in the HP35670A analyser

during the calculation of RMS and RMS exponential FRFs and it is computed as:

4.1.1 Average type: off, time or time exponential – In this case, the equation used is:

$$P_{spec} = crtn * L_{spec} * \overline{L_{spec}} \quad (14)$$

This function is the same as ones presented by equations (6) for each channel used, apart from the correction factor mentioned. Averaging is used in the above formula during the computation of the L_{spec} (Linear Spectrum), as it will be seen in section 4.3.

4.1.2 Average type: RMS – In this case, the formula used is:

$$P_{spec} = crtn * \sum(L_{spec} * \overline{L_{spec}}) / N \quad (15)$$

Here, equation (6) is calculated by taking the averages during the computation of the P_{spec} (Power Spectrum).

4.1.3 Average type: RMS exponential – In this case, the formula used is:

$$P_{spec} = crtn * X_{avg}(L_{spec} * \overline{L_{spec}}) \quad (16)$$

X_{avg} in this case is given by equation (12). The difference here from equation (15) is how averaging is performed.

4.1.4 Average type: Peak Hold – In this case, the formula used is:

$$P_{spec} = crtn * \max(P_{spec}, L_{spec} * \overline{L_{spec}}) \quad (17)$$

4.2 Power Spectral Density

The Power Spectral Density (PSD) is a function which provides power normalised to a 1 Hz bandwidth [8]. This function is useful for wide-band, continuous signal. It displays the response in units squared divided by the equivalent filter bandwidth. So, PSD is nothing more than the P_{spec} values presented from equation (14) to (17) squared divided by the window bandwidth used.

$$PSD = P_{spec}^2 / BW \quad (18)$$

4.3 Linear Spectrum

This function is used during the calculation of the time or time exponential averaged FRF and it can be computed as follows:

4.3.1 Average type: off, RMS, RMS exponential or peak hold – In this case, the formula used is:

$$L_{spec} = crtn * FFT(Wtime) \quad (19)$$

The calculations performed here are the S_x or S_y terms used in equation (4), apart from the correction factor already mentioned.

4.3.2 Average type: time – In this case, the formula used is:

$$L_{spec} = \sum(crtn * FFT(Wtime)) / N \quad (20)$$

As for the case of RMS Power Spectrum (eq. (15)), normal averaging is performed here.

4.3.3 Average type: time exponential – In this case, the formula used is:

$$L_{spec} = \frac{1}{N} (crtn * FFT(Wtime)) + \frac{N-1}{N} * L_{spec} \quad (21)$$

Now, exponential averaging is performed using equation (12), as presented by equation (21).

4.4 Cross Spectrum

This function is used during the computation of the FRF curves using RMS averaging and involves the two signals measured, in the following way:

4.4.1 Average type: off, time or time exponential – In this case, the formula used is:

$$C_{spec} = L_{spec_x} * \overline{L_{spec_y}} \quad (22)$$

Equation (22) is the same as equation (9).

4.4.2 Average type: RMS – In this case, the formula used is:

$$C_{spec} = crtn_x * \overline{(crtn_y)} * \frac{\sum(L_{spec_x} * \overline{L_{spec_y}})}{N} \quad (23)$$

Equation (23) is the same as equation (9), apart from the correction factor already mentioned. The difference from equation (22) is that average is taking during the C_{spec} calculation in this case.

4.4.3 Average type: RMS exponential – In this case, the formula used is:

$$C_{spec} = crtn_x * \overline{(crtn_y)} * X_{avg}(L_{spec_x} * \overline{L_{spec_y}}) \quad (24)$$

X_{avg} in this case is given by equation (12). The difference here from equation (23) is how averaging is performed.

4.4.4 Average type: Peak hold - In this case, no FRF calculation is performed.

4.5 FRF (or transfer function)

As mentioned previously, transmissibility is obtained experimentally by measuring the Frequency Response Function (FRF) between two points of the structure, more or less in the same way FRF curves are obtained during modal testing. Therefore, instead of using the FRF nomenclature in this case, it is used the transmissibility symbol (Tr). Depending on the type of averaging used during the signal process for the FRF calculation, different formulations are employed by the analyser, as shown below. So, the user has to be aware of that.

4.5.1 Average type: off, RMS or RMS exponential - the FRF calculation is performed according to the following equation:

$$Tr_{x,y} = \frac{X}{Y} = \frac{Cspec_{x,y}}{Pspec_y} \quad (25)$$

4.5.2 Average type: time or time exponential - Using this type of average, makes the calculation:

$$Tr_{x,y} = \frac{X}{Y} = \frac{Lspec_x}{Lspec_y} \quad (26)$$

4.5.3 Average type: Peak hold - In this case, no FRF calculation is performed.

4.5.4 FRF calculation final considerations: If one ignores the signal processing performed during the transmissibility calculations and take only the ratio of two measured signal, it can be seen by analysing equations (25) and (26) that, only when the time average is used, the response function obtained will be the same as the mentioned ratio. That is the main objective here.

4.6 Coherence

Coherence is calculated by the HP35670A analyser as follows:

$$Coeh = \frac{Cspec_x * \overline{Cspec_y}}{Pspec_x * Pspec_y} \quad (27)$$

Comparing equations (27) and (10), they are the same and are only computed when performing FRF calculations as presented by equation (25).

5. RESULTS

In order to illustrate the points mentioned previously, several results will be shown to demonstrate the care the user has to have during the measurement of the transfer functions.

A motor operating at 3600 RPM (i.e., 60 Hz), was employed as the source of vibration. Two accelerometers were used to obtain the transmissibility of the motor between its structure and its surrounding surface. So, the output response is measured by the accelerometer connected to the surrounding (in this case, a metallic base where the motor was supported) and the input response is measured by the accelerometer connected to the top of the motor (considered the source of vibration). Both accelerometers were fixed at the same direction.

Initially, three different measurements were performed using different types of averages: 1) time, 2) time exponential and 3) RMS. For each of the measurements, the response for each accelerometer was recorded using the available signal process options on the HP35670A analyser (as in equation (1), using the signal processing presented in items 4.1 and 4.3) and the results obtained from the ratio of the two signals were compared with the results given by the FRF performed by the analyser (and presented in item 4.5).

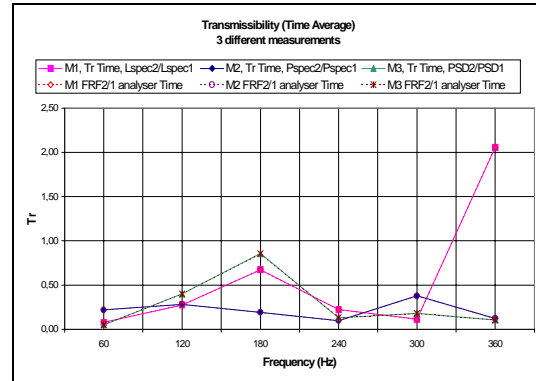


Figure 2 – Calculated and measured transmissibilities, 3 different measurements, Time Average

Figure 2 shows the results obtained for the three measurements using time averaging. It can be seen that the results obtained from the ratio and the FRF measurement are the same for each

one of the measurements. That was expected when looking at equation (26). However, when comparing the results obtained from each measurement, the responses vary a lot. The third measurement gave a transmissibility ratio of more than 2 (two) at 300 Hz, indicating an amplification of the input signal, what may not be true.

When comparing the same type of results but using time exponential averaging (Figure 3), it is demonstrated that the results also varied a lot from one measurement to the next. However, the computation of the ratio and the FRF produced the same result, as expected and proved by equation (26). For the measurements presented in Figure 3, no amplification was detected. However, that may not be a conclusion as the time or time exponential averaging results vary a lot depending on the number of averages performed. That is an important conclusion.

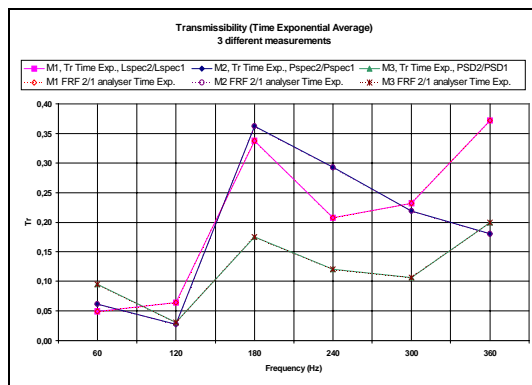


Figure 3 – Calculated and measured transmissibilities, 3 different measurements, Time Exponential Average

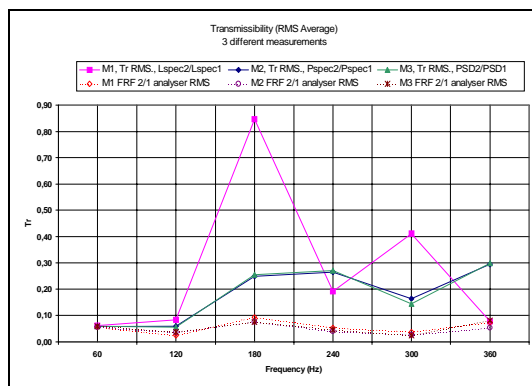


Figure 4 – Calculated and measured transmissibilities, 3 different measurements, RMS Average

The results from the ratio and the FRF calculations proved to be different now when using RMS averaging as shown in Figure 4. That is also expected and can be understood by analysing equation (25). The measured FRF is actually not the ratio of the two measured channels, yet, the ratio between the cross to power spectra. The three different FRF measurements produced approximately the same results, although the measured ratios varied a lot. So, the first conclusion to be drawn is that RMS averaging should be used for transmissibility measurements.

So, emphasis is given next to the measurements performed using the RMS averaging. As mentioned in item 2.2, frequencies at which the vibration energy is concentrated are indicated in the spectrum of the signal as peaks. Figure 5 shows the Power Spectrum Density (PSD) measured for each channel demonstrating that for the motor used, energy is concentrated at 60, 180 and 300Hz. The behaviour shown is the same as that obtained for the Power Spectrum (not shown), although for the PSD representation, the energy is more clearly seen.

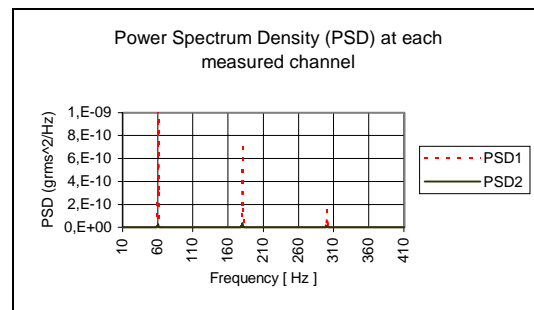


Figure 5 – Power Spectrum Density for each measured channel

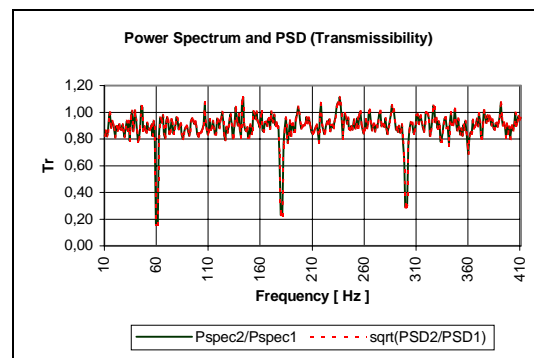


Figure 6 – Power Spectrum and Power Spectrum Density (PSD) Transmissibilities at whole frequency range, RMS averaging, Hanning Window

When transmissibility is calculated as the ratio of the two measured signals, as also mentioned in item 2.2, at frequencies where the energy are concentrated (as seen in Figure 5), the transmissibility values cancel out and the plot becomes flat (see Figure 6). As the other regions are mainly those with zero values, when they are divided by each other, the values may be large (i.e., bigger than 1). However, these values cannot be understood as amplifications of the input signal, due to noise.

As mentioned in item 2.2, the advantage of using RMS averaging is that it is possible to obtain the coherence as well, as a check on the quality of the results. It is only possible to compare results where coherence values are close to unity, as in the other regions, noise may contaminate the quality of the measurements. Moreover, only when the excitation frequency is within the range $0 < \omega < \sqrt{2}\omega_h$ or conversely, when $\omega_h > \omega/\sqrt{2}$ it is possible to have an amplification of the input signal. So, the results will be analysed only at the turning speed and its harmonics.

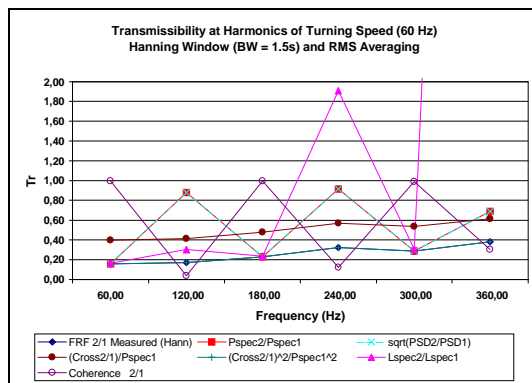


Figure 7 – Transmissibility Functions at Turning Speed, Hanning window, RMS Averaging

Figure 7 presents the results when comparing the several calculations possible using the available measurements. It represents RMS averaging results for the above case using Hanning Window. It can be seen by analysing Figure 7 that where coherence is low, each computation produced a different result, apart from the PSD and Pspec computation that are in fact the same. When using the ratio of the linear spectrum of each channel produced big values for Tr (i.e., $Tr = 1.9$ at 240 Hz and $Tr = 16.8$ at 360 Hz), what may discommend this type of calculation. Nevertheless, it should be stressed that these frequencies had low

coherence. At all high coherence values, the calculations produced almost the same results.

6. CONCLUSIONS

As it can be concluded from the results presented previously, transmissibility should be obtained from continuous systems, using RMS averaging. Although it was not shown here, the number of averages taken during the averaging process is important. It should be great in order for the results to be statistically coherent (around 110 averages). Moreover, only at frequencies where coherence is close to unity that the results can be trusted. In this case, it does not really matter if the calculation is performed as the ratio of the two measured signals or as the normal FRF calculation (i.e., using the cross to power spectra).

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